



# Calculus Cheat Sheet

## Derivatives

### Definition and Notation

If  $f$  is a function of  $x$ , then the derivative is defined to be  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ .

If  $f$  is a function of  $x$ , then all of the following are equivalent notations for the derivative.

$f'(x)$     $\frac{df}{dx}$     $\frac{dy}{dx}$

If  $f$  is a function of  $x$ , all of the following are equivalent notations for derivative evaluated at  $x$ .

$f'(x)$     $\frac{df}{dx}$     $\frac{dy}{dx}$

### Interpretation of the Derivative

If  $f$  is a function of  $x$ , then,

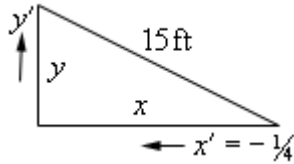




**Related Rates**

Sketch picture and identify known/unknown quantities. Write down equation relating quantities and differentiate with respect to using implicit differentiation ( add on a derivative every time you differentiate a function of ). Plug in known quantities and solve for the unknown quantity.

**Ex.** A 15 foot ladder is resting against a wall. The bottom is initially 10 ft away and is being pushed towards the wall at  $\frac{1}{4}$  ft/sec. How fast is the top moving after 12 sec?



$x'$  is negative because  $x$  is decreasing. Using Pythagorean Theorem and differentiating,

$$y^2 + x^2 = 15^2 \quad 2y y' + 2x x' = 0$$

After 12 sec we have  $x = 10 - 12 \cdot \frac{1}{4} = 7$  and

so  $y = \sqrt{15^2 - 7^2} = \sqrt{176}$ . Plug in and solve for  $y'$ .

$$7 \cdot \frac{1}{4} + \sqrt{176} \cdot y' = 0 \quad y' = -\frac{7}{4\sqrt{176}} \text{ ft/sec}$$

**Ex.** Two people are 50 ft apart when one starts walking north. The angle  $\theta$  changes at  $0.048$  rad/sec. At  $t = 0.24$  sec, how fast is the angle  $\theta$  between them changing when  $\theta = 0.6$  rad?

## Integrals Definitions

**Definite Integral:** Suppose  $f$  is continuous on  $[a, b]$ . Divide  $[a, b]$  into  $n$  subintervals of width  $\Delta x$  and choose  $x_i^*$  from each interval.

Then  $\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x = \int_a^b f(x) dx$ .

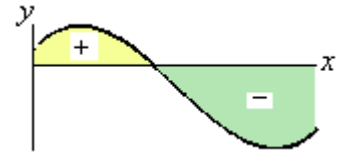




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**Applications of Integrals**

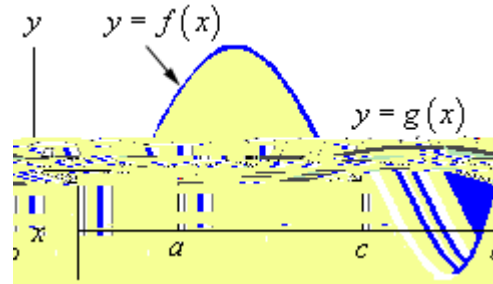
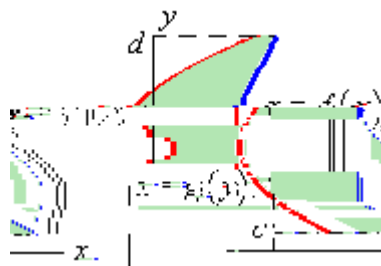
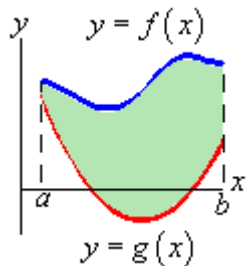
**Net Area :** represents the net area between and the -axis with area above -axis positive and area below -axis negative.



**Area Between Curves :** The general formulas for the two main cases for each are,

upper function lower function  $\int_a^b (f(x) - g(x)) dx$  right function left function  $\int_c^d (f(y) - g(y)) dy$

If the curves intersect then the area of each portion must be found individually. Here are some sketches of a couple possible situations and formulas for a couple of possible cases.



**Volumes of Revolution :** The two main formulas are and . Here is some general information about each method of computing and some examples.

**Rings**

$\pi \int_a^b (outer\ radius^2 - inner\ radius^2) dx$

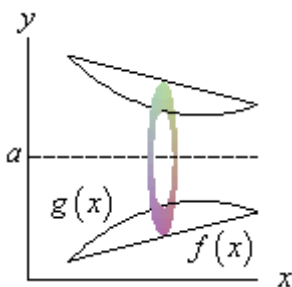
Limits: / of right/bot ring to / of left/top ring  
 Horz. Axis use , Vert. Axis use ,  
 , and . , and .

**Cylinders**

$2\pi \int_a^b radius \cdot width/height dy$

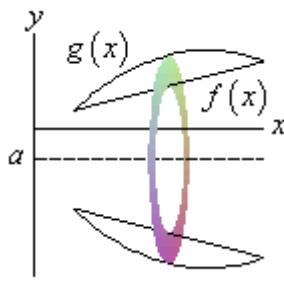
Limits : / of inner cyl. to / of outer cyl.  
 Horz. Axis use , Vert. Axis use ,  
 , and . , and .

**Ex. Axis :** 0



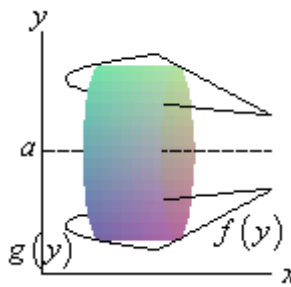
outer radius :  
 inner radius :

**Ex. Axis :** 0



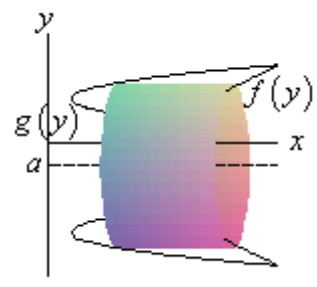
outer radius: ||  
 inner radius: ||

**Ex. Axis :** 0



radius :  
 width :

**Ex. Axis :** 0



radius : ||  
 width :

These are only a few cases for horizontal axis of rotation. If axis of rotation is the -axis use the 0 case with 0. For vertical axis of rotation ( 0 and 0) interchange and to get appropriate formulas.

